Incommensurability and Vagueness - From Values to Probabilities

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Two items are *incommensurable* in value

iff

neither is better than another nor are they equally good.

Is incommensurability possible?

For items within the same value category?

Ruth Chang: *The Small Improvement Argument*

x = Mozart $\quad$ y = Michelangelo.

Intuitively, (i) x is not better (as an artist) than y nor vice versa.

Let $x+$ be a slightly-improved-Mozart:
(ii) $x+$ is better than x.

But, intuitively,
(iii) $x+$ is not better than y.

(iv) If two items are equally good, anything better than one must be better than the other.

(ii) & (iii) & (iv) $\Downarrow$ (v) x and y are not equally good.
(i) & (v) $\Downarrow$ (vi) x and y are incommensurable.

Chang: Mozart and Michelangelo are on **a par**.

Cf. de Sousa (1974), Broome (1978), Sinnott-Armstrong (1985), Raz

BUT: As it stands, this argument does not take into account potential **knowledge gaps** regarding values.
Perhaps we simply don't know (or even cannot know) whether *x* is better than, worse than, or equally as good as *y*.

NOR does the argument take into account potential **vagueness** in value comparisons.
(I am here assuming that vagueness ≠ knowledge gap, pace Williamson.)

We are not willing to say
that *x* is better than *y*, that it is worse, or that they are equally good.

But perhaps this is so only because it is *indeterminate* (= vague)
which of these three value relations obtains between *x* and *y*?

Then it might still be determinate that one of them does obtain,
i.e., it might be determinate that *x* and *y* are *not* incommensurable.

Broome: If vagueness is allowed, then incommensurability not only becomes difficult to *establish*.
Its very *existence* (within one and the same ontological and evaluative category) becomes questionable.

In other words: *Vagueness crowds out incommensurability*.

Broome provides an argument for this claim, but this argument has been criticized by several people.

My objectives:
- To provide a general account of value and a modelling of value relations that makes room for incommensurability

- To show how this modelling can make room for vagueness, along with incommensurability.

Fittting-Attitudes Analysis of Value (FA-analysis)

Valuable = fitting to be favoured
[valuable = desirable, preferable, …]

‘favoured’ – place-holder for a pro-attitude
‘fitting’, ‘appropriate’, “correct”, “warranted”, ‘ought’, “required”,
“reasons”, etc. - the normative component in the analysis.

Brentano, Ewing, Scanlon (Buck-Passing Account of Value)

For *better*, the relevant pro-attitude is thought to be **preference**:

An item x is *better* than y iff it is required to prefer x to y.

Value analysis and incommensurability

In *Ethics* 2004, **Joshua Gert** analysed betterness on FA-lines:

x is better than y iff it is required to prefer x to y.

Gert's main innovation:
The normative component in FA-analysis might have a *stronger* or a *weaker* form.

Requirement *vs* Permission   Ought *vs* May

Introducing two normativity levels makes room for incommensurability.

In particular, one can now define Ruth Chang's notion of **parity** in value, which might be viewed as a prime example of incommensurability:

x and *y* are on *a par* iff it is

(i) permissible to prefer *x* to *y*, and

(ii) permissible to prefer *y* to *x*.

(Gert's own definition of parity was not quite like this. It was more demanding, but wrongly so.)

The analysis can also be extended (going beyond Gert's account) to accommodate *incomparability*
- a radical form of incommensurability.
(For Chang, items that are on a par are comparable.)

Incomparability

For some pairs of items, one might neither prefer one to the other or be indifferent.

It is possible to be in a state of a *preferential gap*.

Preferential gap 🕒 indifference.

In case of a gap,
the situation is typically viewed as being internally conflicted:

Reasons on both sides, but the agent cannot (or does not) balance them off.

An alternative way of thinking of a preferential gap:

Rather than as a mental state in which preferential attitudes are absent,

Incomparability, cont'd

Two items are *incomparable* iff it is not permissible to prefer one to the other or to be indifferent.
What's required is a preferential gap.

Are incomparabilities <u>possible</u>?

Certainly, between items from different ontological categories.
(Say, *x* is a state of affairs and *y* is a person.)
Or between items from different evaluative categories.
(Say, *x* is evaluated as an artist while y as an engineer.)

But what if *x* and *y* belong to the same category?

It may be *permissible* to exhibit a preferential gap regarding *x* and *y*.
[*weak incomparability*]

But can it ever be *required*?

Sophie's choice?

Gert's interval model

The strength of one's preference for an item can be measured.
[interval scale?]

The range of rationally permissible preference strengths with respect to an item *x*:
[*xmin*, *xmax*].

Example: 40 Permissible to prefer *x* to *y*
x 30 and vice versa (parity)

10 *y*
5

Range Rule: x is better than y iff *xmin* > *ymax*.

I.e. iff the weakest permissible preference for *x* is stronger than the strongest permissible preference for *y*.

Objections to the interval model

(i) Incomparability can't be modelled.

If the intervals for *x* and *y* don't overlap, preference for one of the items is required.
And if they do overlap, even at one point, indifference is permissible.

(ii) Certain *betterness structures* can't be represented

Australia + $100 *x* + *y* + South Africa + $100

Australia x y South Africa

So, what's gone wrong?
Is the weakest permissible preference for *x*+ stronger than the strongest permissible preference for *x*? Surely not! Instead, the situation seems to be like this:

x+

x

Intersection modeling

We consider a domain **I** of items
and the class, **K**, of *permissible preference orderings* of that domain.

The orderings in **K** need not be representable by cardinal measures of preference strength.
They might not even be complete rankings: g*aps* are allowed.

However, **K**-orderings are at least well-behaved, in this sense:
preference-or-indifference is a quasi-order (transitive and reflexive)
⇓ Permissible preference is transitive.
And so is permissible indifference.

Intersection modeling – cont'd

Betterness is the *intersection* of preferences in **K**:

x is *better* than *y* iff *x* is preferred to *y* in every ordering in **K**.

x and *y* are
equally good iff they are equi-preferred in every **K**-ordering,

incomparable iff every **K**-ordering contains a gap as regards *x* and *y*,

on a par iff
x is preferred to *y* in some **K**-orderings
and dispreferred to *y* in some **K**-orderings.

And so on.

The Australia - South Africa example:

<u>*P1*</u>	<u>*P2*</u>	<u>*P3*</u>
$x+$	$y+$	$x+$ $y+$
x	y	x y
$y+$	$x+$	
y	x	

Does this model add anything to the original analysis?
Very little.
Which is ok.

But: Formal properties of value relations can now be derived from formal requirements on permissible preference orderings.

Ex.: The transitivity of betterness and of equal goodness follows from the transitivity of permissible preference and equi-preference.

Taxonomy of binary value relations

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
>	+			+		+	+	+	+	+		+			
<		+			+	+	+	+	+		+		+		
=			+	+	+		+	+		+	+			+	
/								+	+	+	+	+	+	+	+
	B	**W**	**E**			**P**	**P**	**P**	**P**						**I**

Injecting *vagueness* into the model (Rabinowicz 2009)

We allow class *K* of permissible preference orderings to be 'unsharp'.
I.e.: *K* might have a number of admissible sharpenings.

➔ It might be indeterminate for some pairs of items what value relation (out of 15 possible) that obtains between them.

Example: Suppose that the domain of items = $\{A, B, C\}$

Consider preference orderings (gappy orderings excluded, for simplicity):

<u>*P1*</u>	<u>*P2*</u>	<u>*P3*</u>	<u>*P4*</u>	<u>*P5*</u>		two sharpenings of *K*:
C	*C*	*C*		A C	*A*	$K1 = \{P1, P2, P3\}$
A	*B*		*A B*	B	*C*	$K2 = \{P1, P2, P3, P4, P5\}$
B	*A*			*B*		

Determinate that (i) *C* better than *B*, (ii) *A* incommensurable with *B* (parity), (iii) *A* not better than *C*.
Indeterminate whether (i) *C* better than *A*,

From values to probabilities

On the FA-approach to value,

to be valuable is to be desirable.

(If we use ”desire” as a stand-in for a pro-attitude.)

We might take a similar approach to probability:

to be probable is to be credible.

Or, to make it slightly more precise:
A proposition *A* is *probable to degree k* iff *A* ought to be given credence of degree *k*.

Ought – relatively to available evidence.

Like the FA-approach to value, this analysis of probability contains an *attitudinal* component (credence) and a *normative* component (ought).

It makes probability an explicitly normative notion.

Thus, it differs from pure subjective probability accounts which identify probability with credence.

Subjective accounts are normative only implicitly: Credences are supposed to be subject to constraints (probability axioms, reflection principle, …)

This FA-approach to probability goes back to the proponents of the epistemic concept of probability:

Poisson (1937),

Cournot (1943),

De Morgan (1847),

Donkin (1851),

Boole (1853),

Jevons (1873),

and, not least, **J. M. Keynes**, *A Treatise on Probability*, 1921.

Keynes (1921)

"The Theory of Probability ... is concerned with the degree of belief which it is *rational* to entertain in given conditions, and not merely with the actual beliefs of particular individuals, which may or may not be rational." (Keynes, p. 3)
"The terms *certain* and *probable* describe the various degrees of rational belief about a proposition which different amounts of knowledge authorise us to entertain." (ibid, p. 2)
"A *definition* of probability is not possible, unless it contents us to define degrees of the probability-relation by reference to degrees of rational belief. We cannot analyse the probability-relation in terms of simpler ideas." (ibid., p. 7)

For Keynes, this FA-account is combined with objectivism about what is rational to believe given evidence:
"When once the facts are given which determine our knowledge, what is probable or improbable in these circumstances has been fixed objectively, and is independent of our opinion." (ibid., p. 3)
But the FA-approach as such does not presuppose objectivism.
It is compatible with subjectivism and with non-cognitivism.

Probability relations – by analogy with value relations

A is *more probable* than *B* iff *A* ought to be given higher credence than *B*.

A and *B* are *equiprobable* iff *A* and *B* ought to be given equal credence.

A and *B* are *incommensurable* (probabilitywise) iff neither is more probable than the other nor are they equiprobable.

A and *B* are *on a par* (probabilitywise) iff
(i) it is permissible to give *A* higher credence than *B*
and
(ii) it is permissible to give *B* higher credence than *A*.
[cases of ambiguous evidence]

A and *B* are *incomparable* (probabilitywise) iff it is impermissible to give higher credence to any of these propositions or to give equal credence to both. [I.e., credence gap is required.]

Weak incomparability = credence gap is permissible.

Interval modelling?

Modelling probability relations by assigning to each proposition *A* an interval of permissible credences [*Amin*, *Amax*],
and by representing the relation *more probable than* by a Range Rule:

A is more probable than *B* iff $Amin > Amax$,

leads to trouble:

(i) incomparability can't be modelled (just as in the case of value)

(ii) some "more probable than"-structures can't be represented:

$A \curlyvee C \quad B \curlyvee C$

A *B*

Let *A* and *B* be propositions that are on a par, probabilitywise.
And let *C* be some proposition that deals with independent matters and is much less probable than both *A* and *B*.

Source of the trouble: Interval modeling lacks resources to specify *permissible combinations* of credences.

Intersection modeling

The class K is now a set of *permissible doxastic states* (permissible given the available evidence).

A doxastic state S is represented as a non-empty set of credence functions on the underlying Boolean algebra of propositions.

Each credence function C that belongs to some S in K is assumed to satisfy Kolmogorov axioms.

If a set S of credence functions is a singleton, then the doxastic state S is determinate.

If it is not a singleton, then S is at least partly indeterminate.

A is assigned *higher credence* than B in S iff for every C in S, $C(A) > C(B)$.

A is assigned *equal credence* as B in S iff for every C in S, $C(A) = C(S)$.

Taxonomy of binary probability relations

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
>	+			+		+	+	+	+	+		+			
<		+			+	+	+	+	+		+		+		
=			+	+	+		+	+		+	+			+	
/								+	+	+	+	+	+	+	+
	MP	**LP**	**EP**			**Par**	**Par**	**Par**	**Par**						**I**

The table gives all the logically possible types of probability relations.
But some such types might not be instantiated.
We might reduce the number of ’realistic’ types
by (i) restrictions on permissible doxastic states,
or by (ii) restrictions on K.

Example of (i):
No doxastic indeterminacy: All permissible doxastic states are singletons.

This would exclude credence gaps and thus types 8-15.

Example of (ii):
Convexity: If in some S in K, A is assigned higher credence than B,
and in some other S in K, B is assigned higher credence than A,
then in some S in K, A and B are assigned equal credence.

This would exclude types 6 and 9.
But one should be very cautious with introducing such restrictions!

A restriction that doesn't reduce the number of types, but is worth considering:

Doxastic Indeterminacy:
If *S*, *S'* *K*, then for some *S''* *K*, *S* *S'* *S''*.

This restriction goes in the direction directly opposite to *No doxastic indeterminacy*.

It makes room for a lot of doxastic indeterminacy in the model.

Injecting vagueness into the intersection model

We do it in just the same way as for value relations:
by making class K 'unsharp',
i.e., by allowing different admissible sharpenings of K.

Thereby, for some propositions, it might become indeterminate what probability relation obtains between them.

Vagueness of probability (= probabilistic indeterminacy) and incommensurability of probabilites are two different phenomena.

Example: $C1(A) > C1(B)$, $C2(A) < C2(B)$, $C3(A) = C3(B)$.

Case 1. $K = \{\{C1\}, \{C2\}, \{C3\}\}$
No indeterminacy. A and B are determinately incommensurable.

Case 2. Three admissible sharpenings of K:
$K1 = \{\{C1\}\}$, $K2 = \{\{C2\}\}$, $K3 = \{\{C3\}\}$.
No incommensurability. A and B are determinately commensurable, but

Note that probabilistic indeterminacy $\neq$ doxastic indeterminacy.

The two kinds of indeterminacy are mutually logically independent.
We can have one without the other. And we can have both or neither.

Doxastic indeterminacy, if present, applies to the elements of K, i.e. to doxastic states.

It makes credence gaps (i.e. vagueness in credence comparisons) permissible.

(This allows for incomparability, but is not needed for incommensurability.)

Probabilistic indeterminacy, if present, depends on K having different admissible sharpenings.

It makes vagueness in probability comparisons possible.

Probability values

Our model allows us to assign 'probability ranges' to every proposition.

If there is no doxastic indeterminacy in the model, i.e., if every permissible doxastic state is a single credence function. Then the probability range of A is the set of numerical values that are assigned to A by permissible credence functions.

If doxastic states are allowed to be indeterminate, then we take as the probability range of A the set of sets of numerical values, such that each of these sets consists of credences assigned to A in some permissible doxastic state.

BUT: These probability ranges cannot be used to define the relation "more probable than".

Thus, suppose (for simplicity) that there are no doxastic indeterminacies and consider two propositions A and B. Suppose their respective probability ranges are intervals. It may well be that the two intervals overlap but B still is more probable than A. (Cf example given earlier, with $B = A \veebar C$.)

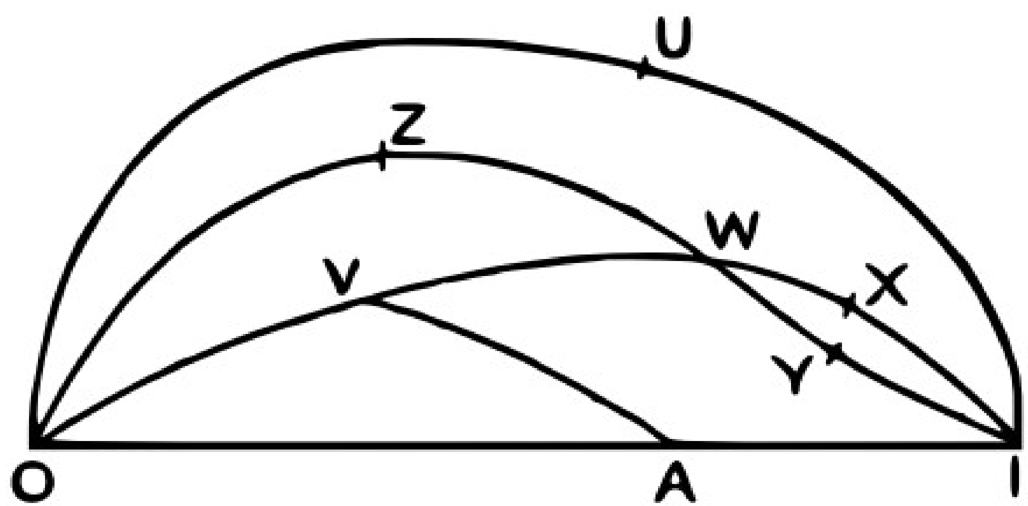
Keynes' claims about probabilities

"I maintain ... that there are some pairs of probabilities between the members of which no comparison of magnitude is possible; that we can say, nevertheless, of some pairs of relations of probability that the one is greater and the other less, although it is not possible to measure the difference between them; and that in a very special type of case ... meaning can be given to a numerical comparison of magnitude." (Keynes, p. 36)

"By saying that not all probabilities are measurable, I mean that
it is not possible to say of every pair of conclusions, about which we
have some knowledge, that the degree of our rational belief in one

Keynes' diagram (ibid. p. 42)

- O is impossibility (probability 0), I is certainty (probability 1).
- Propositions on the same path are linearly ordered by the relation "more probable than".
- Propositions that do not lie on the same path are probabilisticaly incommensurable.
- The same proposition can lie on several paths (example: W).
- Only propositions on the path OAI have numerical probabilities.
- A proposition with a non-numerical probability can lie between propositions with numerical probabilities. (example: V)



Keynes' claims are underwritten by our model:
(i) The model allows of pairs of propositions that are incommensurable probabilitywise.
(ii) It allows a proposition to be more probable than another proposition, even though "it is not possible to measure the [probability] difference between them".
(iii) There is no guarantee in the model that every proposition can be assigned a numerical probability value (either a single number or an interval).
(iv) Still, the model allows, in special cases, an assignment of a pointwise probability value to a proposition (if all credence functions in all permissible doxastic states assign that value to this proposition).
(v) It allows that a proposition which lacks a numerical probability value still has a degree of probability that lies between two numerical values. (At least, its probability will always lie between 0 and 1.)

Main difference:
Keynes doesn't seem to allow *optionality* in rational degrees of belief::
When he actual degrees of belief that are not uniquely rationally determined by evidence, he considers them to be "arbitrary" (which suggests that they are unjustified).

"Consider, for instance, the reinsurance rates for the *Waratah* , a vessel which disappeared in South African waters. The lapse of time made rates rise; the departure of ships in search of her made them fall; some nameless wreckage is found and they rise; it is remembered that in similar circumstances thirty years ago a vessel floated, helpless but not seriously damaged, for two months, and they fall. Can it be pretended that the figures which were quoted from day to day— 75 per cent, 83 per cent, 78 per cent—were rationally determinate, or that the actual figure was not within wide limits arbitrary and due to the caprice of individuals?" (Keynes, p. 24)

It's sometimes ok to *act* according to "caprice", but that's a different matter:
"Is our expectation of rain, when we start out for a walk, always more likely than not, or less likely than not, or as likely as not? I am prepared to argue that on some occasions none of these alternatives hold, and that it will be an arbitrary matter to decide for or against the umbrella. If the barometer is high, but the clouds are black, it is not always rational that one should prevail over the other in our minds, or even that we should balance them,—though it will be rational to allow caprice to determine us [i.e. To determine our action, I take it – W.R.] and to waste no time on the debate."
(p. 31?

That there is no room for optionality was explicitly asserted by Harold **Jeffreys**, *Scientific Inference*, Cambridge: CUP 1931.

But Jeffreys had no room for unmeasurable or incommensurable probabilities either.
The probability of a proposition given data is on his view uniquely determined.

“On a given set of data *p* we say that a proposition *q* has in relation to these data one and only one probability. If any person assigns a different probability, he is simply wrong .... Personal differences in assigning probabilities in everyday life are not due to any ambiguity in the notion of probability itself, but to mental differences between individuals, to differences in the data available to them, and to differences in the amount of care taken to evaluate the probability” (p. 10)
The same view is repeated in his *Theory of Probability*, Oxford: Clarendon Press, 1939, where he talks about “the unique reasonable degree of belief” (p.39) [quoted after Galavotti 2011]

Points for discussion

Should we relativize probability not just to evidence, but also to a *prior* (a prior credence assignment, or more generally a prior doxastic state)? This seems reasonable.

But the prior might involve a particular commitment as to how to respond to evidence, which means that the optionality in response is decreased in this way.

Still, optionality need not be totally eliminated by the prior doxastic state and this is all we need in order to be able to distinguish between two levels of normativity.